Acoustic-Instability Boundaries in Liquid-Propellant Rockets: Theoretical Explanation of Empirical Correlation

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Introduction

ACOUSTIC instabilities in liquid-propellant rocket engines are phenomena in which oscillations of the chamber pressure at well-defined frequencies and mode shapes that correspond to the acoustic modes of the cavity are amplified mainly through interactions with combustion.[1,2] This type of instability can be most destructive to rocket engines and most difficult to predict and control. Predicting acoustic instabilities continues to pose many difficulties, mainly because of lack of understanding the highly complex and nonlinear nature of turbulent combustion processes. Although there have been a few predictive models, traditional strategy employed to eliminate acoustic instability has often relied on trial-and-error of a large number of full-scale tests. For example, during the development of the F-1 engine of the Saturn V rocket, over 2000 full-scale tests were conducted from October 1962 to September 1966 (Ref. 3). Present-day economic constraints do not allow such a large number of full-scale tests to achieve the goal, and better predictive models are required to minimize the number of full-scale tests.

In practice, empirical methodologies based on the previous engine-test results have often been used to estimate where the boundaries of acoustic instability lie. One of such empirical approach is the Hewitt stability correlation,[4] in which the stability boundary of liquid-propellant rocket engines employing impinging-jet injector designs is related to the acoustic frequencies and a parameter determining the injector operating condition. The Hewitt stability correlation has demonstrated good agreements in matching the stability characteristics of more than 20 full-scale engines used for production or for technology programs. Figure 1 shows the correlation for rocket engines using liquid oxygen (LOX)/RP-1 propellant combinations and like-on-like impinging-jet injectors, in terms of the highest sustainable acoustic frequency and the injection parameter d_0/U_i, where d_0 is the orifice diameter of the injector and U_i is the injection velocity of the RP-1. The correlation is shown for rocket engines with a wide range of chamber diameters, including the 1.0-m-diam F-1 engine (5U, PFRT, and Qual); the 0.53-m-diam H-1 engines; a 0.2-m-diam engine used in a U.S. Air Force combustion-instability technology program (-0100); a 0.14-m-diam engine used in a technology program at the NASA Lewis Research Center (LeRC Pavli); and 90-mm- and 0.14-m-diam engines used in a Marshall Space Flight Center heavy hydrocarbon technology program (HHC H-1 derivative and HHC canted fan).

Figure 1 demonstrates a high-frequency cutoff, that is, the highest sustainable frequency is inversely proportional to the injection parameter. The 5U was the earlier baseline injector design for the F-1 engine, which was typically unstable at 750 Hz. Modifications to this injector eventually resulted in the marginally stable PFRT injector design and the stable Qual injector design. The H-1 engines were also tested as a part of the F-1 stability development effort. The H-1 engine with 0.73-MN thrust, which was stably operating, was throttled to determine the thrust level at which instability commences. Instabilities with frequencies up to 3000 Hz were observed at a thrust level in excess of 0.89 MN. Additional Hewitt stability correlations for different propellant combinations and injector arrangements are presented in the paper by Anderson et al.[4] and in the references therein. This Note addresses the question of explaining from fundamentals why the stability boundary in Fig. 1 is as observed.

Counterflow Flamelet Response Model

Based on the Rayleigh criterion,[5] it has long been understood that combustion with a suitable phase response is the main physical process capable of providing the energy necessary to amplify and sustain acoustic waves in rocket engines. In the first attempt to identify the main acoustic instability mechanisms, the response characteristics of various flames to acoustic waves were analyzed[6,7] for time-dependent vaporization and combustion in droplet and laminar-jet diffusion flames to calculate their amplitude and phase relationships. A comprehensive summary of this work was given by Crocco.[8] However, all of these early flame-response studies were carried out with infinite rate chemistry, thereby precluding effects coming from the interaction of the chemical kinetics with the acoustic waves. Since the dominant nonlinear term in combustion processes arises from the chemical reaction, neglecting the finite rate chemical-reaction effects may lead to a significant underestimation of the amplification rate.

The introduction of full, detailed chemical kinetics makes analyses exceedingly complicated. Therefore, as a mathematical model for the chemical mechanism, activation-energy asymptotics (AEA) has recently been employed.[9,10] In the limit of asymptotically large Zel'dovich number β (a measure of the ratio of the activation energy to the thermal energy), chemical reaction is confined to an asymptotically thin layer having a thickness of order β^{-1} compared to the diffusive transport zone, and the characteristic time for reaction is much shorter, by order β^{-2}, than the characteristic diffusion time. For flames near extinction, the characteristic diffusion time of laminar diffusion flamelets can be estimated as the extinction strain rate, which for hydrocarbon/oxygen flames is in the range of 10^{-3} to 10^{-4} s, of the same order as the characteristic acoustic time given by the reciprocal of the acoustic frequency. Therefore, the time scale of the acoustics in the application is comparable

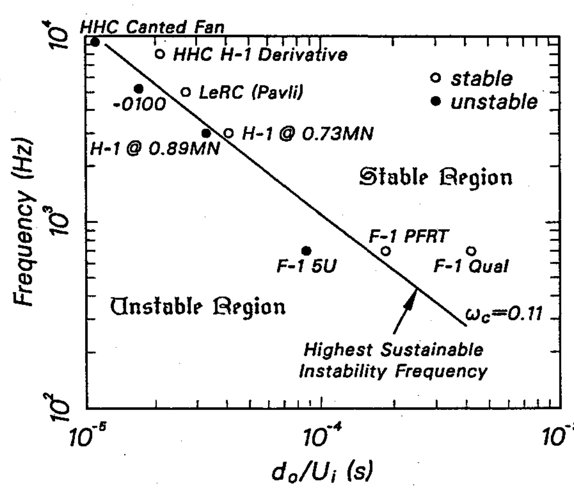

Fig. 1 Hewitt stability correlation plotted in terms of the highest sustainable instability frequency as a function of d_0/U_i for rocket engines using LOX/RP-1 and like-on-like injector.

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to that of flamelet diffusive transport, such that the outer diffusive–convective layer is modified to include the unsteadiness caused by the acoustic waves, while the inner reactive–diffusive layer remains quasisteady. The effect of finite rate chemistry will then influence the unsteady flame response through the instantaneous matching conditions at the reaction sheet.

In our recent work,[10] this AEA flame analysis has been carried out for strained laminar diffusion flames as a model flamelet in liquid-propellant rocket engines to predict the response of the reaction sheet and the burning rate to the imposed acoustic pressure oscillations in terms of their magnitude and phase. These counterflow flamelets are viewed as existing between the fuel and oxygen spray fans. The high sensitivity near extinction can be seen from the well-known S-curve behavior. Figure 2a, obtained from a calculation performed in a counterflow diffusion flame,[10] shows a typical result of the steady reaction-sheet response as a function of the Damköhler number Da, which is a ratio of the characteristic flow time (reciprocal strain rate) to the characteristic chemical time. It is seen that, for large Da, the steady reaction sheet is close to the Burke–Schumann limit, so that the flame temperature is close to the adiabatic flame temperature, but as Da decreases by increasing the strain rate, the flame temperature continuously decreases and finally exhibits a turning point that corresponds to static extinction. It is clear from Fig. 2a that, for the same fractional variation of Damköhler number $\delta Da/Da$, shown by vertical bars, the near-extinction flame experiences a much larger reaction-sheet displacement, and the flame-temperature variation, as marked by horizontal bars, is also much larger.

More detailed results of the calculation are shown in Figs. 2b and 2c in terms of $\tilde{h}$, a complex nondimensional rate of heat-release perturbation per unit reaction-sheet area and per unit reactant consumed. The flame gives an amplification contribution if the real part of h is positive. The nondimensional amplification at a given nondimensional acoustic frequency $\omega = 1$, where ω is the acoustic frequency divided by the strain rate, is seen in Fig. 2b to be gradually increasing as the flame approaches extinction, so that $Re(h)$ is larger by an order of magnitude near extinction than near equilibrium. From Fig. 2c, $Re(h)$ is also found to be monotonically decreasing with increasing ω because larger diffusion-layer accumulation of the unsteadiness associated with faster acoustic oscillations tends to impede the flame response. In view of these results, it can be concluded that the flame response is anticipated to be larger at least by an order of magnitude when flames are near extinction and when the characteristic acoustic time is longer than the characteristic flow time in the flamelet.

Even greater importance of the near-extinction flame response to acoustic amplification is attributed to the higher acoustic-pressure amplitude in the near-injector region where most of these flamelets exist. The higher amplitude is caused by the nonuniform acoustic medium. The distribution of acoustic pressure in nonuniform media is described by a generalized Helmholtz equation for the amplitude φ, $\nabla\cdot(\nabla\varphi/\rho) + \lambda^2\kappa\varphi = 0$, where ρ is the density, κ is the adiabatic compressibility defined as $\kappa \equiv 1/\rho a^2$, and a is the frozen sound speed. The nondimensional frequency λ is an eigenvalue of the generalized Helmholtz equation. If ρ and κ are constant, the classical Helmholtz equation is recovered. Solution for the generalized Helmholtz equation can be obtained by a variational principle that states that a functional defined by $\sigma \equiv \int \rho^{-1}\nabla\varphi\cdot\nabla\varphi\, dx/\int \kappa\varphi^2\, dx$ is minimized by an acoustic eigenfunction, and the local minimum of σ with respect to the variation of φ corresponds to λ^2. If flows in rocket engines are assumed to be a mixture of ideal gases and liquid spray, then there exists a significant decrease in the density toward the downstream end of the combustion chamber because of progress of the vaporization and combustion processes, while the adiabatic compressibility is found to be nearly constant because the chamber volume is mostly occupied by ideal gas, the adiabatic compressibility of which is a constant. To minimize the functional σ with the density decreasing downstream, the acoustic pressure gradient is larger in the high-density region (near injectors). Consequently, the amplitude of the acoustic pressure is higher in the near-injector region.[11] This type of acoustic-pressure profile was reported experimentally already in early work by Levine,[12] in which the variation was so pronounced that the measured pressure signal at the entrance of the nozzle section showed negligible pressure fluctuations.

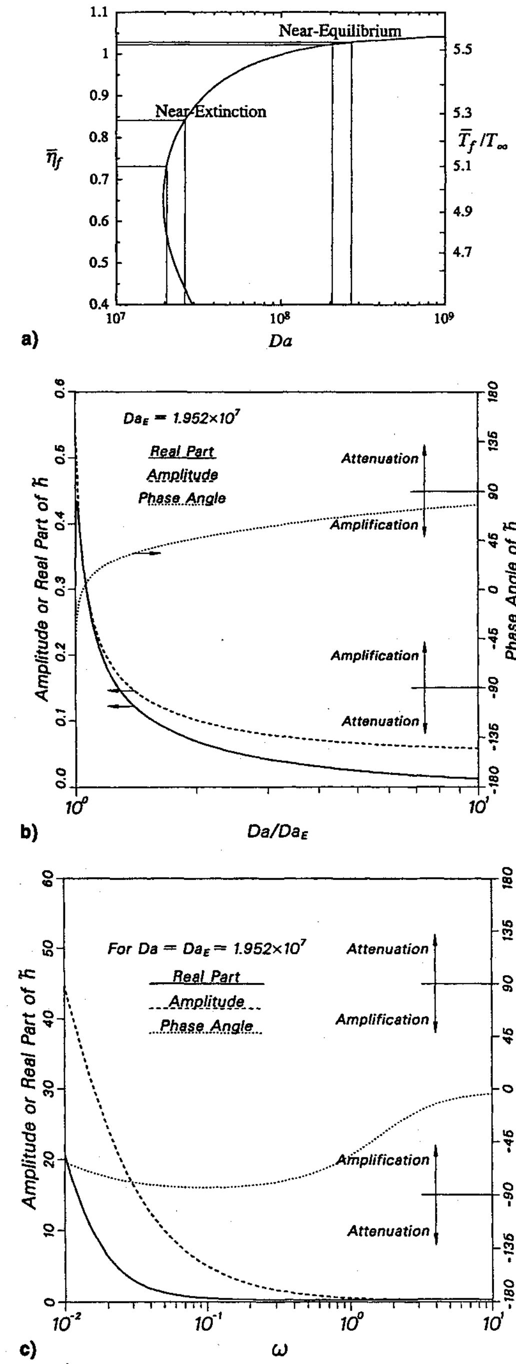

Fig. 2 Characteristics of flame responses near-extinction: a) steady reaction-sheet location and flame temperature as a function of the Damköhler number, showing the high sensitivity of the flame response near extinction; b) dependence of the $\tilde{h}$ on the Damköhler number for $\omega = 1$; and c) dependence of $\tilde{h}$ on ω.

In liquid-propellant rocket chambers, the rate of strain decays in the axial direction, so that flamelets near extinction may be expected to be encountered more often in the near-injector region. Because the amplification contribution is proportional to the combustion response function with a weighting function φ^2, the dominant amplification effect is considered to be coming from flamelets near the injectors. This is consistent with observations in engine tests, indicating that stability is often achieved by moving the flame zone away from the injector assembly.[1,3] The effects of finite rate chemistry are also consistent with the temperature-ramping test, a stability-rating method in which the fuel is gradually cooled until instability begins to appear (or, more often, heated until it disappears). As the temperature of the reactants increases, equilibrium chemistry becomes more prevailing, and the consequent reduction of the amplification contributions according to Fig. 2a results in stable operation of the rocket engine, according to this viewpoint.

Explanation of Amplification Correlation

To identify the instability mechanism responsible for the Hewitt stability correlation, Anderson et al.[4] conducted a characteristic-time analysis, which shows that jet breakup and droplet shattering exhibits relatively similar trends to the correlation. It was therefore suggested that atomization response is a key instability mechanism. To test this claim, the characteristics of the impinging-jet spray were investigated for a range of the injection parameter d_0/U_i. From the experiments, the mean droplet diameter was found to decrease with decreasing d_0/U_i, which favors faster vaporization and combustion responses. However, atomization response alone is unlikely to drive acoustic instabilities, because sufficient energy to sustain the acoustic oscillations is likely to be available only from the combustion process. Although atomization affects the subsequent combustion, the atomization response does not necessarily produce combustion response with a correct phase relationship to amplify the acoustic waves. Atomization could be directly relevant if the time lag between atomization and combustion was sufficiently smaller than the acoustic time, but numerical estimates indicate that this is not so.[2,13] Therefore, it is more attractive to explain the acoustic instability mechanism directly from the flame-response characteristics, for which the relevant flow time is the same injection parameter d_0/U_i, the reciprocal of the characteristic strain rate for the flamelets near the injector assembly.

In attributing the dominant amplification effects to the near-extinction flamelets in the near-injector region, it is desirable to try to establish that such flamelets exist. To date, there are not many conclusive experiments showing whether flames near the injectors are composed of strained diffusion flamelets enveloping the spray or of individual droplet flames. However, photographs taken by Levine[12] appeared to be strained diffusion flamelets surrounding LOX sprays, and a recent visualization of LOX/H_2 flames,[14] although taken in a single-element coaxial-injector test rocket, clearly demonstrates strained diffusion flamelets enveloping the LOX spray jet. The LOX spray appears to be too dense to support individual droplet flames because penetration of the gaseous fuel into the LOX spray is insufficient. The strained diffusion-flamelet model, outlined in the previous section, therefore appears likely to reasonably represent the flamelets found in the near-injector region.

The strain rates for these flamelets can be approximated as U_i/d_0, which is the strain rate at the impact point of the two jets and also is the reciprocal of the injection parameter. Since the strain rate in impinging-jet flows tends to be spatially uniform, the value of U_i/d_0 represents the strain rate for flamelets in a relatively wide region near the injector. However, the strain rate is expected to fluctuate about U_i/d_0 because of turbulence. As the strain rate U_i/d_0 is increased by increasing U_i or by decreasing d_0, the chance to encounter near-extinction flamelets also increases, thereby resulting in a larger amplification rate. This instability tendency is consistent with the Hewitt correlation in that instability appears by decreasing the injection parameter, i.e., increasing the strain rate. As shown in Fig. 2c, the flame response rapidly decreases with increasing acoustic frequency, thereby suggesting a high-frequency cutoff. A substantial increase of the flame response is seen in Fig. 2c if the nondimensional acoustic frequency ω is smaller than a value that is typically less than unity. Therefore, the cutoff frequency should be linearly proportional to the strain rate U_i/d_0, which is again consistent with the Hewitt correlation. Figure 1 suggests that the nondimensional cutoff frequency is about $\omega_c \approx 0.11$, below which a rapid increase of the flame response is apparent in Fig. 2c. Although it is of course necessary to balance amplification against attenuation, and many different phenomena arise, it is interesting that this one particular phenomenon exhibits all of the right functional dependences.

This Note attempts to present a new interpretation of acoustic-instability phenomena found in liquid-propellant rocket engines, based on the results of the flame-response analyses in which the effects of finite rate chemistry are taken into account. The new theory is consistent with many instability attributes observed during engine tests, in particular with the Hewitt stability correlation compiled for rocket engines employing impinging-jet injector designs. However, our lack of understanding of the nature of turbulent combustion processes still hinders further improvements of the qualitative and quantitative prediction methodologies for acoustic instability. There are only a few experiments that were successful in characterizing the turbulent flames in rocket engines, and most of them were conducted in test rockets employing coaxial injectors. Visualization of turbulent flames stabilized near impinging-jet injectors is needed, but have yet to be accomplished. This absence of knowledge in conclusive flame characterization leaves rather large uncertainties. Finally, it should be re-emphasized that acoustic instability has not been encountered without combustion, so that attempts to identify the instability mechanisms should include the combustion-response characteristics along with the other transport processes.

Acknowledgments

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